$$a_{sh} = -F\frac{a_{10}^2 R_{10}}{\rho_{10}}, \quad F = \frac{M_{30}(2\gamma M_{10}^2 + 1 - \gamma) - (\gamma - 1)(M_{10}^2 - 1)}{M_{30}(2\gamma M_{10}^2 + 1 - \gamma) + 2M_{10}^2 + \gamma - 1}$$

where M_{10} is the Mach number of the shock wave, M_{30} is the Mach number behind the wave front. We can show that $F > 0$ for $M_{10} > 1$. Therefore the sign of a_{sh} is minus the sign of R_{10}, i.e., if the density ahead of the wave increases, the shock wave decelerates, and conversely. The absolute value of the acceleration increases as the square of M_{10}.

5. In conclusion we note the following. The problem of the breakup of an ordinary discontinuity plays an important role in numerical methods of the mechanics of continuous media. In particular, its solution is used to construct finite-difference schemes for the numerical integration of the unsteady equations of gas dynamics (so-called Godunov-type schemes /3/). The result is a numerical scheme of first-order approximation, which leads to certain errors in numerical calculations. The analytical solution of the generalized Riemann problem obtained in this paper may be used to improve the order of approximation of the Godunov scheme if the piecewise-constant approximation is replaced by a piecewise-linear approximation.

REFERENCES

1. BEN-ARTZI M. and FALKOVITZ J., A second-order Godunov-type scheme for compressible fluid dynamics, J. Comput. Phys., 55, 1, 1984.
2. KOCHIN N.E., On the theory of discontinuities in a fluid, in: Collected Papers, 2, Izd. Akad. Nauk SSSR, Moscow-Leningrad, 1949.
3. GODUNOV S.K., ZABRODIN A.V., IVANOV M.YA., et al., The Numerical Solution of Multidimensional Problems of Gas Dynamics, Nauka, Moscow, 1976.

Translated by Z.L.

J. Appl. Maths Mechs, Vol. 55, No. 1, pp. 74-78, 1991
Printed in Great Britain

0021-8928/91 $15.00+0.00

ON THE POSSIBLE MODES OF FLOW ROUND TAPERED BODIES OF FINITE THICKNESS AT ARBITRARY SUPERSONIC VELOCITIES OF THE APPROACH STREAM*

A.I. RYLOV

Supersonic planar flow round a symmetric tapered body is considered for which, at each point, the angle of inclination of the wall is less than the limiting angle for the shock polar corresponding to the approach stream. It is shown that states of flow with the formation of both an attached shock wave (SW) of the strong family and a detached SW with subsequent subsonic flow between the shock wave, the body and the sonic line are impossible at any stream velocities. In essence, the results obtained by Nikol'skii /1/ are transferred to the case of an arbitrary Mach number of the approach stream.

The impossibility of flow round a finite wedge with the formation of an attached SW of the strong family has been proved when substantial simplifying assumptions are made in /2/. The problem has been considered in /1/ in a general formulation under the sole assumption that there are no local supersonic zones and closed stream lines in the subsonic flow domain between the SW, the body and the sonic line.

In this paper, the proof is based on a monotonic change in the angle of inclination of the velocity vector along lines of constant pressure (isobars). This fact, which is valid in the case of vortex flows, has been established previously in /1/ and the analogous result for

**Prikl. Matem. Mekhan.*, 55,1,95-99,1991

vortex-free flows /3/ preceded it. As a result, it was shown in /1/ that, at Mach numbers of the approach stream $M_\infty < 1.7$ (the adiabatic index $\varkappa = 1.4$), states of flow round tapered bodies with an attached SW of the strong family and with a detached SW are impossible and, at the same time, it was pointed out that, when $M_\infty > 1.7$, a combined analysis of the subsonic and supersonic flow domains is required. It is possible that this constraint on M_∞ and, this means, also on the limiting angle of inclination of the wall, served as one of the reasons why this interesting result was not echoed in subsequent publications. For instance, in the well known handbooks /4-8/, the question of the possibility or impossibility of the formation of an attached SW of the strong family during the flow round tapered bodies of finite thickness is barely discussed at all and it is merely noted that the SW of the weak family is realized experimentally.

In this paper use is made of a fundamental element in the proof in /1/ which is based on an analysis of an isobar emerging from a corresponding point of the SW, with the refinement of certain details. However, another estimate of the pressure on the sonic line is used rather than that in /1/. As a result, the basic derivations are now independent of the Mach number of the approach stream.

Let us consider the planar symmetric flow round a tapered body of finite thickness by a uniform horizontal supersonic flow of an ideal (non-viscous and non-thermally conducting) gas with a Mach number M_∞ and an adiabatic index $\varkappa$. The upper half Oh of the body being considered is shown in Fig.1. The shock polar corresponding to the approach stream is shown in Fig.2. Here and subsequently, we adopt the following notation: M is the Mach number, q and θ are the modulus and angle of inclination of the velocity vector and $p, \rho,$ and s are the pressure, density and entropy.

The shock polar is symmetrical about the $\theta = 0$ axis. The points c and c^- correspond to the value $M = 1$ while points located above (below) the points c and c^- correspond to values of $M < 1$ $(M > 1)$. At the point k (k^-), the angle θ reaches its limiting maximum (minimum) value θ_k $(-\theta_k)$. It is well-known that $M < 1$ at the points k and k^- in the case of polytropic gases.

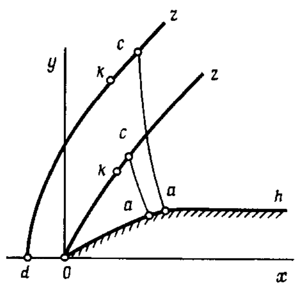

Fig.1

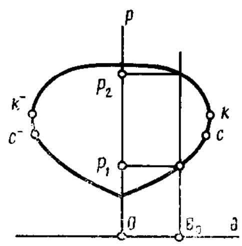

Fig.2

We recall that, in the case of an infinite wedge with a taper angle θ_0 $\theta_0 < \theta_k$ (Fig.2), theory admits of two states of flow with attached shock waves corresponding to the weak($p = p_1$) and strong ($p = p_2$) families where $p_1 < p_k < p_2$. Correspondingly, different states of flow might also be expected in the case of a body of finite thickness depending on its geometry.

The following proposed states of flow round a tapered body Oh (Fig.1) will be considered next.

State 1. Flow with an attached SW Oz which corresponds, at the point of tapering, to the strong family. Here, $M \leqslant 1$ at all points of the domain Oac.

State 2. Flow with a detached SW dz. Here, $M \leqslant 1$ at all points of the domain $dOac$.

The necessary existence of the sonic lines ac is associated with the fact that, upon the unbounded removal of the SW from the body, Oz and dz degenerate into characteristics. Consequently, there are sonic points c on Oz and dz from which sonic lines emerge which either reach the body at point a or which depart to the right along the horizontal wall of the body to infinity. The latter can only take place in the case of a detached SW for a narrow range of initial parameters, M_∞ and $\varkappa$ /9/. In this case, it is sufficient to presume that point a in Fig.1 is displaced to the right to inifinity and that $M < 1$ along the whole of the wall. It is found that, on account of the occurrence of the above-mentioned sonic lines, the problem of the flow round bodies of finite thickness can be fundamentally different from the problem of the flow round an infinite wedge.

It follows from the assumptions which have been made that there are no breaks on the segments Oc and dc and all the parameters, including the entropy s, vary continuously along them, but not necessarily in a monotonic manner. In the regions of subsonic flows, Oac and $dOac$, not only p and θ are continuous but also s, and this means M, q and ρ also.

The investigation of subsonic non-isentropic flows in the regions Oac and $dOac$ was based on an analysis of the isobars /1/. For instance, by using the equations of the gas dynamics of planar vortex flows in the form

$$p_N = -\rho q^2 \theta_L, \quad (M^2 - 1)\, p_L = -\rho q^2 \theta_N$$

the following expression was obtained for the derivative calculated along an isobar:

$$\theta_l = -p_n (1 - M^2 \sin^2 \beta) / (\rho q^2) \tag{1}$$

where p_L, θ_L, p_N, and θ_N are derivatives calculated along the streamlines and along the normal to a streamline, p_n is a derivative calculated with respect to the normal to an isobar and β is the angle between the velocity vector and an isobar.

The following conclusion was drawn on the basis of (1).

If the reduced pressure domain remains to the left ($p_n \leqslant 0$), as one moves along an isobar in a planar subsonic vortex flow, then the velocity vector rotates monotonically (possibly not strictly monotonically) in an anticlockwise direction in the case of such motion.

Using this property of isobars, it has been shown /1/ that the proposed states of flow are impossible when $M_\infty < 1.7$.

For greater justification and effectiveness in the use of the above-mentioned properties of isobars when analysing the subsonic flows behind SW's, we shall prove the following assertions in addition to /1/.

Assertion 1. In the subsonic domain behind a shock wave into which a homogeneous horizontal supersonic flow is incident from the left, the pressure cannot reach its local extremum at points in the shock wave with the exception of the points at which the two conditions: $\theta = 0$, $\theta_y < 0$ are simultaneously satisfied. The second condition means that, behind the shock wave, the flow filament is constricted and the current in it is speeded up. Such a situation can occur, for example, beyond a point of irregular reflection of a shock wave from a plane of symmetry.

In fact, if the pressure in the subsonic domain behind the shock wave has a local extremum at a certain point t of the shock wave, then, in this case, isobars, which begin and terminate in the shock wave, must exist in a small neighbourhood of the point t and encompass t. However, by virtue of the properties which have been mentioned above, this is only possible when $\theta = 0$, $\theta_y < 0$ and, as a consequence of this, it is only in such a case that an isobar does not reach from point t into the subsonic region. In all the remaining cases the isobars reach from points in the shock wave into the subsonic domain. Use will later be made of this fact.

Assertion 2. In the states of flow which are being considered, the maximum pressure on the sonic lines ac is equal to the pressure p_c at the sonic point c of the shock polar.

Actually, an entropy minimum is reached on segments Oc and dc when $M = 1$, that is, at the points c. Consequently, a minimum in the entropy is also reached on ac at the same points. By allowing for the fact that $p = p_0(s) f(M, \varkappa)$, where $p_0(s)$ and $f(M, \varkappa)$ are known functions, and that $p_0(s)$ increases as s decreases, we obtain that $p \leqslant p_c$ on ac, which it was required to prove.

Assertions 1 and 2 provide additional information regarding the extremal properties of subsonic vortex flows. We also recall /1/ that the pressure p cannot attain a local extremum at internal points of the domain of subsonic flow. Points which are encompassed by closed isobars, where the angle θ varies by 2π when they are passed around, are an exception to this. The latter occurs during the collision of streams from a common stagnation point and in the case of closed streamlines /8/.

We shall make use of Assertions 1 and 2 in the proof of the following theorem.

Theorem. The proposed states of flow round a tapered body which have been formulated above subject to the condition that $\theta \leqslant \theta_k$ on the subsonic segment of the wall and $\theta = \theta_0 < \theta_k$ at the point of tapering are impossible at any supersonic velocities of the approach stream.

Proof. By virtue of the continuity of the gas-dynamic parameters along the segments Oc and dc, points must exist on them at which θ reaches the maximum value of θ_k for the shock polar. We shall also denote these points in a shock wave by the index k (if there are several such points, the index k refers to those closest to the points c). Following /1/, let us consider isobars which emerge from these points into the subsonic regions Oac and $dOac$. The existence of such isobars follows from Assertion 1. It is well-known /5/ that branching points are possible in subsonic flows. When there are such points, we choose the outer left branches for the continuation of the isobars investigated (here and subsequently, the motion occurs from the point k).

With such a construction of the isobars, $p_n \leqslant 0$ and here p_n only vanishes at the above-mentioned branching points. Consequently, when $M \leqslant 1$, $\theta_l \geqslant 0$ along the isobars investigated by virtue of (1), and $\theta > \theta_k$ at all points of the isobars other than k. The isobars which have emanated from the points k cannot suddenly terminate within the subsonic domains and they must reach some points on the boundaries of these domains. It follows from Assertion 2 and from the form of the shock polar that $p \leqslant p_c \leqslant p_k$ on the sonic lines and, consequently, the isobars investigated cannot reach the sonic lines ac. However, since $\theta > \theta_k$ on these isobars, they cannot reach the shock wave Oc and dc, the wall Oa and also the interval dO of the plane of symmetry. The resulting contradiction thereby proves the theorem.

Remark 1°. In /1/, the pressure p on the sonic lines was estimated from above by the quantity p^* which is equal to the pressure obtained after the isentropic retardation of the approach stream to $M = 1$. It has been shown /1/ that, for $\varkappa = 1.4$ when $M < 1.7$, we have $p_k > p^*$, on the basis of which the conclusion was drawn regarding the impossibility of the above-mentioned states of flow when $M_\infty < 1.7$.

2°. In proving the theorem, the exotic situation was omitted in which the angle θ along the isobars investigated increases by a significant amount $\varphi \leqslant 2\pi$ and, at the same time, generally speaking, it is possible that an isobar reaches the shock wave, the wall or the plane of symmetry. Without going into detail, we merely note that the lines $\theta = \theta^* = \text{const}$ $(\theta_k < \theta^* < 2\pi)$ which have issued from points of such isobars cannot again reach these isobars and cannot reach the shock wave, the wall of the body and the plane of symmetry. Only their intersection at a certain point t of the subsonic region is possible and, here, the point t must be encompassed by closed isobars where, when they are traversed completely, the angle θ changes by 2π. As has already been mentioned above, the pressure p at such a point t must reach its local extremum. In connection with the problem under consideration, only a flow with closed streamlines as, for example, in the case of leading detached zone, is possible in the neighbourhood of the point t.

3°. The theorem does not extend to the case when an attached shock wave of the weak subsonic family, which corresponds to the points of the arc ck of the shock polar which also includes the case when $\theta_0 = \theta_k$, is realized at the point of tapering.

Actually, the occurrence of points at which the value of θ is not less than the maximum value of θ on the subsonic segment of the wall is not guaranteed in this case on the segment Oc to the right of point O. In its turn, the absence of such points does not enable us to arrive at a contradiction in the analysis of isobars on which the proof of the theorem was based.

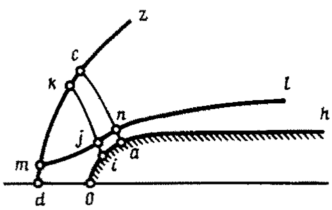

Fig.3

4°. If $\theta = \theta_0 < \theta_k$ at the point of tapering but values of $\theta > \theta_k$ are possible on the subsonic segment of the wall, then the theorem is invalid in this case, which can be seen from the proof of the theorem. Consequently, states of flow with a detached shock wave and with an attached shock wave, corresponding to the strong subsonic family, are possible in this case regardless of the shape of the body. Corresponding examples of flow with a detached shock wave are well-known /7/ and we shall therefore dwell in greater detail on an example of a flow with an attached shock wave of the strong subsonic family.

Let us consider the flow of the blunt body, Oah, in Fig.3. here, dcz is the detached shock wave, ac is the sonic line, k is a point at which $\theta = \theta_k$ and ki are isobars. Then, as the upper half of the required tapered body, one may take any streamline, mnl, which reaches the shock wave from the interval dk. It is obvious that the shock wave in the neighbourhood of point m belongs to the strong subsonic family. Next, $M < 1$ in the domain mnc and $M = 1$ on nc and $\theta > \theta_k$ at point j. In other words, the fact that the Mach number and the angle of inclination of the wall simultaneously increase along the initial part of its wall is a characteristic feature of the body which has been constructed.

5°. If the gas being considered is not polytropic, the theorem remains valid when Assertion 2 is satisfied. If, at the same time, the sonic point c of the shock polar is located above the point k, then θ_k has to replace by θ_* in the condition appertaining to the theorem.

The author thanks A.N. Kraiko, Yu.B. Lifshitz and V.V. Sychev for useful discussions and advice.

REFERENCES

1. NIKOL'SKII A.A., On the planar vortex flows of a gas, in: Theoretical Investigations in Fluid and Gas Mechanics, (Ed. by A.A. Nikol'skii), Tr. TsAGI, 2122, 1981.
2. FRANKL F.I., On the question of the uniqueness of the solution of the problem of supersonic flow round a wedge, Prikl. Matem. i Mekhan., 10, 3, 1946.

3. NIKOL'SKII A.A. and TAGANOV G.I., Motion of a gas in a local supersonic zone and certain conditions for the breakdown of potential flow, Prikl. Matem. i Mekhan., 10, 4, 1946.
4. LANDAU L.D. and LIFSHITZ E.M., Theoretical Physics. 6, Hydrodynamics, Nauka, Moscow, 1986.
5. MIESES R., Mathematical Theory of Flows of a Compressible Fluid, IIL, Moscow, 1961.
6. OVSYANNIKOV L.V., Lectures on the Foundations of Gas Dynamics, Nauka, Moscow, 1981.
7. CHENYI G.G., Gas Dynamics, Nauka, Moscow, 1988.
8. SEDOV L.I., Planar Problems in Hydrodynamics and Aerodynamics, Nauka, Moscow, 1966.
9. TERENT'YEV E.D., On the form of the sonic line accompanying the flow of a supersonic stream of ideal gas round a blunt wedge, Dokl. Akad. Nauk SSSR, 247, 2, 1979.

Translated by E.L.S.

J. Appl. Maths Mechs, Vol. 55, No.1, pp. 78-85, 1991
Printed in Great Britain

0021-8928/91 $15.00+0.00

NON-STATIONARY PROBLEM OF A PLANE HYDRAULIC FAULT CRACK IN A FLUID-SATURATED STRATUM*

YU.N. GORDEYEV

The problem of a vertical hydraulic fault crack /1/ in a fluid-saturated stratum wedged out by a viscous filtering fluid flow is considered. It is assumed that the state of stress and strain of the stratum is described by a system of Biot equations /2/. A system of elastic constant notation, proposed in /3/, is used.

The non-stationary problem of a vertical hydraulic fault crack in a fluid-saturated stratum in one special case of representing the general solution of the consolidation theory equations reduces to the solution of an equation of piezoconductivity type with a source and a formula connecting displacement of the crack edges with the fault fluid pressure and the fluid leakage velocity through the crack walls. In the case of a fixed "ideal" crack, along which the pressure is constant, the problem of a hydraulic fault reduces to solving a one-dimensional singular integral equation for the Laplace transform. Asymptotic forms of the solution of this equation are found for long and short times. Representation of the general solution of the consolidation theory equations in the Papkovich-Neuber form was obtained to solve consolidation theory problem /4, 5/. Compressibility effects of the interstitial fluid /6/ were taken into account in the development of this method. A representation of the general solution of the consolidation theory equations in terms of two analytic functions of a complex variable /3/ was obtained in another approach to the solution of plane problems. Application of consolidation theory to the investigation of stationary problems of a hydraulic fault of a fluid-saturated stratum was started in /7, 8/.

1. Formulation of the problem. Let a plane crack in an infinite porous fluid-saturated space in a homogeneous compressive stress field σ_0 be maintained in an open state by fluid heated within the crack, which can filter through its wall into a porous medium while moving along the crack. It is assumed that the borehole radius r_0 can be less than the crack length L_0 and, consequently, effects associated with the presence of the borehole can be neglected.

In particular, this crack theory problem occurs in connection with the problem of a hydraulic fault in an oil-bearing stratum /1/.

A coupled theory of consolidation /3/ $(i, j, k = 1, 2, 3,$ and summation is over repeated subscripts) is used to describe the strain of a fluid-saturated porous medium and the filtration of the interstitial fluid therein:

$$d\sigma_{ij}/\partial x_j = 0, \quad \sigma_{ij} = \sigma_{ji} \tag{1.1}$$

*Prikl.Matem.Mekhan.,55,1,100-108,1991